* M B V - 0 0 3 - 1 1 6 4 0 0 4 - 1 0 0 - 4 *

MBV-003-1164004

Seat No. ______

M. Sc. (Sem. IV) Examination

April / May - 2018

Mathematics : CMT - 4004

(Graph Theory) (New Course)

Faculty Code : 003

Subject Code : 1164004

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Each question carries 14 marks.

(2) All the question are compulsory.

1 Answer any **seven** from following short questions : **7×2=14**

(i) Define following terms :

Self loop, Parallel edges, null graph and simple graph.

(ii) Define subgraph of a graph. Draw a graph G with its two subgraphs H_1, H_2 so that $V(H_1) \cap V(H_2)$ is a single ton set and $E(H_1) \cap E(H_2)$ = empty set.

(iii) Define closed walk and cycle. Give an example of a closed walk of a graph G which is not a cycle in G.

(iv) Define Eulerian graph. Draw an Eulerian graph G, which is a simple graph but it is not a k-regular graph for any $k \in \{1, 2, ..., |V(G)| - 1\}$.

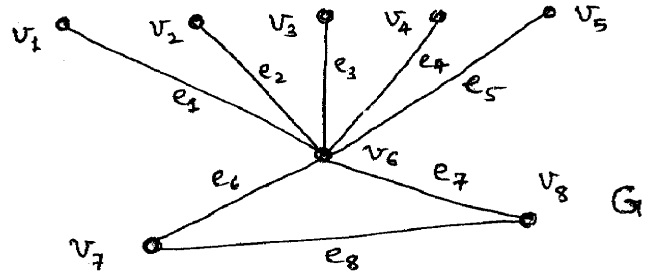
(v) Define Hamiltonian cycle and Hamiltonian graph. Draw a wheel graph Wn with its a Hamiltonian cycle, for some integer $n \geq 3$.

(vi) Draw a simple graph G with following properties :

Number of components of G is atleast three, no component of G is a null graph, $|V(G)| = 9$ and $|E(G)| \leq 7$.

(vii) Define incidence matrix of a self loopness graph G. Also write down the incidence matrix K_3 (Complete graph on three vertices).

(viii) Write down all the spanning trees of following graph G.



2 Attempt any **two** : **2×7=14**

(a) Define connected graph G. Prove that a graph G is disconnected if there are two non-empty disjoint subsets V_1, V_2 of $V(G)$ such that

(1) $V_1 \cup V_2 = V(G)$ and

(2) There is no edge $e \in E(G)$, whose one end vertex lies in V_1 and another end vertex lies in V_2.

(b) For a connected graph G, prove that G is an open Eulerian graph if G has exactly two odd vertices and remaining all vertices are even vertices if exist.

(c) Let G be a graph and it contains exactly two odd vertices say x and y. Let for any $V \in V(G) - \{x, y\}$, $d_G(V)$ = even. Prove that there must be a path in G between x and y.

(d) Let T be a tree. Prove that any two distinct vertices u and v of T, there is a unique path between u and v in T.

3 Attempt any **one** : **1×14=14**

(a) For a connected planner graph G, prove that $f = e - n + 2$. Also derive following :

(1) $e \geq \frac{3f}{2}$

(2) $e \leq 3n - 6$.

Where $e = |E(g)|, n = |V(G)|$ and f = the number of faces in the graph G.

(b) For a connected graph G, prove that the ring sum of two cutsets of G is either a cutset for G or it is an edge disjoint union of two cutsets.

(c) State and prove Eulerian theorem.

4 Attempt any **two** : **2×7=14**

(a) State and prove Max flow min cut theorem.

(b) Prove that Kuratowaski's first graph K_5 and second graph $K_{3,3}$ both are non-plannar graphs.

(c) For a tree T, prove that $|E(T)| = |V(T)| - 1$.

(d) For an acyclic graph G, Prove that $|E(G)| = |V(G)| - k$, where k is the number of components for the graph G.

5 Attempt any **seven** : **7×2=14**

(1) Define minimally connected graph. Draw a minimally connected graph G with $|V(G)| = 4$.

(2) Define eccentry of a Vertex and distance between two vertices in a connected graph.

(3) State the statement of Konig's theorem. Also write down number centers for a path P_6 on six vertices.

(4) Write definitions of fundamental cycle and fundamental cut-set of a graph G.

(5) Define weighted graph and minimal spanning tree.

(6) Draw dual graphs of K_3 and K_4.

(7) Draw a simple graph whose adjacency matrix is given by

$$X(G) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

(8) Write down proper edge coloring and proper vertex coloring of following graph :